

National Research University
Higher School of Economics

Program of entry exam in
in Mathematics

In English

The Syllabus of the Entry Exam in Mathematics

The following syllabus contains two parts.

The first section lists the basic mathematical concepts that applicants should be familiar with.

The second section shows which skills are required from applicants during the exam.

The volume of knowledge and the degree of mastery of the material, which are described in the syllabus, match the appropriate mathematics course at a high school. Applicants may use all concepts from this course, including precalculus. However, in order to solve the exam tasks it is enough to be familiar with the notions and properties, which are listed in this syllabus. Concepts, which are not studied at a secondary school, can be used as well, but the only condition is that the applicant should be able to explain and prove them in the detailed solution.

Free response questions are based on the following principle:

There can be different algorithms applied while solving free response questions. The main requirement is that the solution should be mathematically literate: the reasoning of the author's work should be clear. The rest (method/notation) can be arbitrary. The completeness and validity of the arguments are evaluated regardless of the algorithm of the solution. This assessment is done in such a way that the progress in solving the problem is evaluated rather than the shortcomings in comparison with the "standard" solution.

Entry exam are held in the form of a written test. It consists of 30 problems and the maximum score is 100 points. The duration of the exam is 180 minutes. Different problems score different points ranging from 3 points for a basic problem to 5 points for more complex problems.

The Content of the Entry Exam in Mathematics

I. Basic concepts

1. Algebra

1.1. Numbers, roots and powers

- 1) Integers
- 2) Natural number power
- 3) Fractions, percentages, rational numbers
- 4) Integer power
- 5) Root of degree $n > 1$ and its properties
- 6) Rational power and its properties
- 7) Real power and its properties

1.2. Fundamentals of trigonometry

- 1) Sine, cosine, tangent, cotangent of an arbitrary angle
- 2) Radian measure of an angle
- 3) Sine, cosine, tangent and cotangent of a number
- 4) Basic trigonometric identities
- 5) Cofunction identities
- 6) Sine, cosine and tangent of a sum and of a difference between two angles
- 7) Sine and cosine of a double angle

1.3. Logarithms

- 1) Logarithm of a number
- 2) Logarithm of a product, quotient, power
- 3) Decimal and natural logarithms, the constant e

1.4. Transformation of expressions

- 1) Transformation of expressions including arithmetic operations

- 2) Transformation of expressions involving exponentiation
- 3) Transformation of expressions involving roots
- 4) Transformation of trigonometric expressions
- 5) Transformation of expressions involving logarithms
- 6) Module (absolute value) of a number

2. Equations and Inequalities

2.1. Equations

- 1) Quadratic equations
- 2) Rational equations
- 3) Irrational equations
- 4) Trigonometric equations
- 5) Exponential equations
- 6) Logarithmic equations
- 7) Equivalence of equations, systems of equations
- 8) Trivial systems of equations with two unknowns
- 9) Basic methods of solving systems of equations: substitution, addition, introduction of new variables
- 10) Use of properties and graphs of functions for solving equations
- 11) Solution set of equations with two variables on the coordinate plane
- 12) Application of mathematical methods to solve text problems from different fields of science. Interpretation of the results, taking real-life constraints into account

2.2. Inequality

- 1) Quadratic inequality
- 2) Rational inequality
- 3) Power inequality
- 4) Logarithmic inequality
- 5) Systems of linear inequalities
- 6) Systems of inequalities with one variable
- 7) Equivalence of inequalities, systems of inequalities
- 8) Use of properties and graphs of functions for solving inequalities
- 9) Method of intervals
- 10) Solution set of inequalities with two variables on the coordinate plane

3. Functions

3.1. Definition and graph of a function

- 1) Domain of a function
- 2) Image of a function
- 3) Graph a function. Examples of functional dependencies in real-life processes and phenomena
- 4) Inverse function. Graph of an inverse function
- 5) Transformation of graphs: parallel translation, reflectional symmetry

3.2. Elementary study of functions

- 1) Monotonic functions. Monotonicity intervals
- 2) Even and odd functions
- 3) Periodicity of a function
- 4) Bounded function
- 5) Points of extremum (local maximum and minimum) of a function
- 6) Maximum and minimum values of a function

3.3. Basic elementary functions

- 1) Linear function, its graph
- 2) Rational function, its graph
- 3) Quadratic function, its graph
- 4) Power function with a natural base, its graph
- 5) Trigonometric functions, their graphs
- 6) Exponential function, its graph
- 7) Logarithmic function, its graph

4. Precalculus

4.1. Derivative

- 1) Concept of a derivative of a function, geometric meaning of derivative
- 2) Physical meaning of a derivative, finding speed for the process defined by a formula or a graph
- 3) Equation of a tangent to a graph of a function
- 4) Derivative of a sum, a difference, a product, a quotient
- 5) Derivative of elementary functions
- 6) Second derivative and its physical meaning

4.2. Analysis of a function

- 1) Use of a derivative for plotting functions
- 2) Examples of use of derivatives to find optimal solutions in different applications, including the socio-economic problems

5. Geometry

5.1. Planimetry

- 1) Triangle
- 2) Parallelogram, rectangle, diamond, square
- 3) Trapezium
- 4) Circle and circumference
- 5) Circumscribed and inscribed circle
- 6) Polygon. Sum of the angles of a convex polygon
- 7) Regular polygon. Inscribed circle and circumscribed circle of a regular polygon

5.3. Polyhedra

- 1) Prism, its base, lateral edges, height, side surface, right prism, regular prism
- 2) Parallelepiped, cube, symmetry of a cube, symmetry of a parallelepiped

5.4. Measurement of geometric quantities

- 1) Angle measure, degree of an angle, correspondence between an angle and length of a circular arc
- 2) Area of a triangle, parallelogram, trapezium, sector
- 3) Volume of a parallelepiped, cone, pyramid

5.5. Coordinates and vectors

- 1) Cartesian coordinates on plane and in space
- 2) Formula of a distance between two points, equation of a sphere
- 3) Vector, length of a vector, equality of vectors, addition of vectors and multiplication of a vector by a number
- 4) Collinear vectors. Decomposition of a vector into two noncollinear vectors
- 5) Coplanar vectors. Decomposition of a vector into three non coplanar vectors
- 6) Coordinates of a vector, scalar product, angle between vectors

6. Elements of combinatorics, probability theory and statistics

6.1. Elements of combinatorics

- 1) Consecutive and simultaneous selection
- 2) Number of combinations and permutations. Binomial theorem

6.2. Elements of statistics

- 1) Tabular and graphical data presentation
- 2) Numerical characteristics of series

6.3. Elements of probability theory

- 1) Probability of events
- 2) Use of probability and statistics for solving of applied problems

II. Requirements

At the exam in mathematics the applicant should be able to:

1. Be able to perform calculations and transformations:

- A) To perform arithmetic operations, combining oral and written techniques; find values of a root of a natural power, a rational power, a logarithm;
- B) To calculate the values of numerical and algebraic expressions, performing the required transformations and substitutions; to calculate approximate expressions;
- C) To calculate algebraic expressions, applying formulas and rules of transformation for powers, radicals, logarithms and trigonometric functions.

2. To be able to solve equations and inequalities:

- A) To solve rational, irrational, exponential, logarithmic and trigonometric equations and systems of these equations;
- B) To solve equations, trivial systems of equations, using the properties of functions and their graphs; to use graphical method to find approximate solutions of equations and inequalities;
- C) To solve rational, exponential and logarithmic inequalities and systems of these equations.

3. To be able to perform operations with functions:

- A) To determine the value of a function, depending on its argument when functions are defined in different ways; to describe behavior and properties of a function, using the graph; to find maximum and minimum values, using a graph; to plot graphs of functions;
- B) To calculate derivatives of trivial functions;
- C) To explore monotonicity of functions in trivial cases, to find maximum and minimum values of a function.

4. To be able to perform operations with geometric figures, coordinates and vectors:

- A) To solve planimetric problems, related to finding lengths, angles and areas;
- B) To determine the coordinates of a point; to perform operations on vectors, to calculate length of a vector and angle between vectors.

5. To be able to construct and explore trivial mathematical models:

- A) To model real-life situations, using algebra; to translate into equations and inequalities information given in a task; to explore models, using algebra;

- B) To model real-life situations, using geometry; to explore models, using geometric concepts and theorems and algebra; to solve practical problems related to finding geometric quantities;
- C) To prove statements, to evaluate logics of the reasoning, to recognize illogical reasoning;
- D) To model real-life situations, using probability theory and statistics; to calculate probability of events in trivial cases.

6. To be able to use acquired knowledge and skills on practice and in everyday life:

- A) To analyze real data and statistical information; to calculate, using formulas; to use estimates in practical calculations;
- B) To express real-life relationships between values, using functions, and to interpret their graphs; to extract information provided in tables and graphs;
- C) To solve applied problems, including problems concerning socio-economics and physics, that require to find maximum and minimum values, to find speed and acceleration.