Entry exam on mathematics A sample test of 2016

The test consists of 30 problems. You have 180 minutes.

1.What is the number of solutions of the system of equations $\begin{cases} xy = 6, \\ x + y = 5 \end{cases}$?

Answer: 2

2.A ton of grapes has a humidity of 99%. While stored it was dried out and the humidity became 98%/ What is the mass of the dried grapes?

Answer: 500kg

3.Find the product of all the solutions of the equation $\sin(x+1) = 0$ that lie within the interval $-5 < x < 3$.

Answer: $\pi^2 - 1$

4.Find the range of the function $x + \sin x$ defined on $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$.

Answer: $\left[-\frac{\pi}{2} - 1, \frac{\pi}{2} + 1\right]$

5.Evaluate expression $\frac{a^4 - b^4}{a^3 - b^3} \cdot \frac{a^2 + ab + b^2}{a + b}$ at $a = \sqrt{3}, b = \sqrt{7}$.

Answer: 10

6.Find the least value of the function $\log_3(x^2 - 4x + 13)$ on the closed interval $[0;5]$.

Answer: 2

7.Find the range of the function $y = 4^{\sin x}$ which is defined on $[\frac{\pi}{6}; \frac{2\pi}{3}]$.

Answer: $[2;4]$

8.Find the number of the roots of the equation $\sqrt{2}\sin^3 x - \sqrt{2}\sin x + \cosh 2x = 0$ that lie within the closed interval $[-\frac{5\pi}{2}; \pi]$.

Answer: 8

9.A good is produced in two different qualities: quality 1 and quality 2. Initially one unit of quality 2 good has cost 10% less than the one unit of quality 1 good. Due to the seasonal changes quality 2 good firstly became 10% more expensive and later became cheaper by 20%. Quality 1 good firstly became more expensive by 10% and later became cheaper by 10%. Compare final prices of the goods. By what percentage the quality 1 price is higher that the quality 2 price?

Answer: 25%

10.Sergei had put a deposit in a bank at 10% annual percentage rate. After full 3 years he withdrew the money in the amount of 15972 rubles. What was his initial deposit?

Answer: 12000

11.In a right triangle ABC with the hypotenuse BC the lengths of the legs are $AB = 5$ and $AC = 12$. Find the length of the perpendicular drawn from the vertex A to the hypotenuse.

Answer: $4\frac{8}{13}$

12.Find all the values of the parameter $k > 0$ for which the system $\begin{cases} y = \sqrt{k - x^2}, \\ xy = 1 \end{cases}$

has a unique solution.

Answer: $k = 2$

13.Sum up all the integer numbers that lie within the domain of the following function $y = \sqrt{\frac{4 - x}{(x-1)|x-3|}}$.

Answer: 6

14.Find the difference of the arithmetic progression $a_1, a_2, ...$given equations $a_1 + a_2 + a_6 = 5.1$ and $a_3 + a_5 + a_7 + a_9 = 8$.

Answer: 0.1

15.Find the number of the roots of the equation $\sin(\pi x) = \frac{3}{2}\tan^2(\pi x)$ that belong to the interval $(0.5; 1.5)$.

Answer: 1

16.A water tank has the shape of a rectangular parallelepiped and is made of steel sheets of the fixed thickness. The area of the base is 0.16 m^2 and the volume is 80 litres. Find the dimensions of the lateral sides in order to minimize the mass of the empty tank.

Answer: in metres: 0.4 times 0.4 times 0.5

17.Two pipes are used to empty the pool. If two pipes are open then the pool will be completely emptied in 20 hours. If the first pipe is open but the second is not the water would flow out in 25 hours. Find what time will be needed to empty the pool if the second pipe is open but the first is not.

Answer: 100 hours

18.Find the integer part of the biggest root of the equation $x^3 - 10x^2 + 17x = 0$.

Answer: 7

19.Find the remainder left after dividing the sum of all the roots of equation $|x^2 - 12x + 27| = x$ by 5.

Answer: 4

20.Find all the values of the parameter a for which the system of equations $\begin{cases} xy = a, \\ 2x + b^2 y = 3 \end{cases}$ has at least one solution for all the values of parameter b.

Answer: $a \le 0$

21.In a regular quadrangular pyramid the area of a side surface is twice as much as the area of the base. Find the ratio of the height to the length of the side of the base of the pyramid.

Answer: $\sqrt{\frac{3}{5}}$

22.Find the area of an octagon whose vertices are the solutions of the system of equations $\begin{cases} |x| + |y| = 7, \\ x^2 + y^2 = 25 \end{cases}$ in $xy-$plane.

Answer: 62

23.Find $x_1^2 + x_2^2$ where x_1, x_2 are roots of the equation $x^2 + 4x - 3 = 0$.

Answer: 22

24.Count the number of the integer solutions of the inequality $\log_2(5 - 4x - x^2) < \log_2(x^2 + 7)$.

Answer: 4

25.Find the value of the derivative of the function $y = \sin(3x) + x^2 + 2x - 1$ at the point $x = 0$.

Answer: 5

26.Find all the values of the parameter a for which equation $ax^2 + 2\sqrt{2}x - (a+3) = 0$ has a unique solution.

Answer: $a \in \{-2; -1; 0\}$

27.There are two jars with the acid solutions. The first jar is twice as big as the second. The first jar contains 15% solution, the second jar contains 30% solution. Both jars are emptied into a big jar and the solution is stirred. Find the percentage of the solution in this big jar.

Answer: 20%

28.Count the number of the solutions of the equation $\left|x^2 - 6x - 23\right| = 7$.

Answer: 4

29.A merchant sells 3 feet of the red cloth and 4 feet of the green cloth for 31 dollars or he sells 4 feet of the red cloth and 3 feet of the green cloth for 32 dollars. How much will he charge for 14 feet of the red cloth and 11 feet of the green cloth?

Answer: 114

30.Given a system $\begin{cases} |xy| = M, \\ (x-1)^2 + (y+1)^2 = 4 \end{cases}$, find the biggest possible number of its solutions, where parameter M may take all real values.

Answer: 8